A method to approximate Hadamard finite part transforms on the positive semiaxis

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Denote by \( H_p(f, t) \) the \( p \)-Hadamard Finite Part Transform of a given function \( f \)

\[
H_p(f, t) = \int_0^{+\infty} f(x) \frac{w_{\alpha, \beta}(x)}{(x-t)^{p+1}} dx, \quad t > 0,
\]

\[
w_{\alpha, \beta}(x) = e^{-x^\beta} x^\alpha, \quad \alpha \geq 0, \quad \beta > \frac{1}{2}, \quad 1 \leq p \in \mathbb{N}.
\]

The approximation of \( H_p(f) \) is of interest in different contexts. For instance, in the solution of hypersingular integral equations coming from Neumann 2D elliptic problems on a half-plane (see [1]). To our knowledge, most of the papers available in the literature deal with the approximation of Hadamard integrals on bounded intervals (see for instance [3], [4] and the references therein). In [1], the case of unbounded intervals is reduced to bounded ones by means of suitable transformations. Finally, in [2] we have proposed a method for approximating the integral in (1) for any fixed \( t \) by means of a suitable truncated Gauss-Laguerre rule. However, when “many” values of \( H_p(f, t) \) have to be computed, it can be more convenient the procedure we go to describe.

Setting

\[
H_p(f, t) = \sum_{k=0}^{p} \frac{f^{(k)}(t)}{k!} \int_0^{+\infty} \frac{w_{\alpha, \beta}(x)}{(x-t)^{p+1-k}} dx,
\]

we approximate the function

\[
F_p(f, t) := \sum_{k=0}^{p} \frac{f^{(k)}(t)}{k!} \int_0^{+\infty} \frac{w_{\alpha, \beta}(x)}{(x-t)^{p+1-k}} dx
\]

by means of the derivatives of a suitable truncated Lagrange polynomial interpolating \( F_0(f, t) \) at the zeros of the \( m \)-th Generalized Laguerre polynomial
$p_m(w_{\alpha,\beta})$ (see [5]). The procedure employs the truncated Gauss-Laguerre rule and uses the interlacing properties of the zeros of the Generalized Laguerre polynomials. The error estimate in a weighted uniform norm is proved and some numerical tests confirming the efficacy of the proposed procedure will be shown.

References