Mathematical models for solving the Shallow Water Equations applied to the study of urban flooding

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Keywords. Finite volume methods, shallow water equations, urban flooding models

The Shallow Water Equations (SWE) are a time-dependent system of non-linear partial differential equations of hyperbolic type. They can be obtained from the depth-averaged Navier-Stokes equations and express the conservation laws of mass and momentum. The SWE can be applied in several problems of practical interest, involving free-surface water flows under the influence of gravity. Flood propagation in rivers and in the neighbouring areas is a typical example for which the use of the SWE is required. The numerical modelling of flood inundation, especially in urban areas, is becoming a key issue in flood risk assessments due to its ability in the prediction of possible flooding scenarios, giving the opportunity to formulate suitable flood hazards mitigation measures. The reason of this increasing interest lies in the unacceptable costs in terms of properties damage and even human lives that usually occur when an urban area is affected by flooding. Except ideal cases, these equations cannot be solved analytically and for this reason several numerical methods have been proposed in the literature. In this context, widely used approaches to solve the SWE are the so called Godunov-type Finite Volume Methods [5,6], which allow one the computation of numerical fluxes. In fluvial engineering applications, the presence of irregular topographies poses further numerical difficulties that are also tackling in the literature in order to obtain stable and reliable results. In particular, attention has to be paid to the numerical treatment of the source terms of the SWE in order to avoid spurious oscillations due a bad balance scheme for pressure and gravity forces. For this reason, the numerical integration of the source term should be carried out in agreement with the flux functions. Another numerical challenging problem concerns the management of small water depths near wet/dry interfaces that require specific algorithms in order to avoid numerical instabilities. In urban areas, the numerical solution of the SWE, written according the two horizontal directions, is complicated by the presence of buildings and other man-made structures. Probably, the most accurate technique to describe the effects of buildings is to consider them like no submergible obstacles, imposing internal solid wall boundary conditions. In this
way, it is possible to simulate the flood propagation through the buildings but, due to the huge number of computational cells, this technique might not be feasible for the numerical computations in wide areas using a common computing machine. In order to overcome this problem, an alternative technique is based on a modification of shallow water equations introducing a porosity coefficient to represent constriction effects and the volume occupied by obstacles [3]. Apart from the techniques used to take into account the presence of an urban areas, particularly important is the type of the computational grid used to discretize the physical domain. For example, the use of an unstructured grid allows one to modify the density of the grid points accordingly to the topographic features and the expected physical situations. This grid has a higher degree of flexibility and adaptability [2], especially around buildings, than that of a structured grid but is more difficult to manage. The main purpose of this work is the compared analysis of two numerical models to deal with flooding in urban areas. All the numerical schemes considered in this work belong to the family of finite volume methods. In particular, the first one uses the Roe scheme for the computation of the numerical fluxes. It is based on an unstructured grid, composed of irregular triangular elements, and the effects of buildings are modeled imposing internal boundary conditions [1]. The second one is based on the HLLC Riemann Solver, implemented on a structured mesh, and applied to the modified version of the two dimensional SWE, introducing the porosity concept. In both the models, particular attention has been paid to the numerical treatment of source terms in order to provide a balance with the numerical fluxes. The numerical models have been applied for the numerical simulation of experimental tests carried out within the European Project IMPACT (Investigation of extreme flood Processes And unCerTainty) with particular reference to the so called "Flash flood experiment in a simplified urban district" [4].

References