A dual-mixed finite element method for non-Newtonian fluid flow problems

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We propose a dual-mixed formulation for non-Newtonian fluid flow where the fluid viscosity is assumed to be nonlinear function of the rate of strain tensor. The governing equations arise in modeling flows of, for example, biological fluids, lubricants, paints and polymeric fluids. In [3,4] we have introduced and analyzed a dual-mixed finite element method for quasi-Newtonian fluid flow obeying to the power law. A priori error estimates for the finite element approximation were proved in [3] while a posteriori error estimation was provided in [4]. However, in both [3,4] the analysis used the assumption that the equation describing the stress tensor in terms of the rate of strain tensor was invertible to give the rate of strain tensor as function of the stress tensor. The mixed finite element method developed in [3] possesses local (i.e., at element level) conservation properties (conservation of the momentum and the mass) as in the finite volume methods. Furthermore, it allows the approximations of all the physical variables (stress, velocity and pressure).

The aim of this work is to extend our investigations by avoiding the assumption of expressing the rate of strain tensor as function of the stress tensor. As example of such a situation is a non-Newtonian fluid flow obeying the Carreau law. For this purpose, we introduce an additional variable for the rate of strain tensor and reformulate the governing equations as a twofold saddle point problem. It must be noted that this kind of approach has been introduced and analyzed in [5,2] for a class of quasi–Newtonian Stokes flows. However, in both [5,2] the tensor gradient of the velocity was used instead of the rate of strain tensor. The fact to use the rate of strain tensor introduces a major difficulty in the construction of mixed finite element methods (for more details, see [1]). The difficulty lies essentially in the symmetry of this tensor. One way to overcome this difficulty is to relax the symmetry of this tensor by a Lagrange multiplier. We will present our dual-mixed formulation and establish well posedness. The mixed finite element for this formulation will be provided and the associated a priori error estimates are then derived.
References


