Effortless construction of hierarchical spline quasi-interpolants

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Quasi-interpolation is a well-known technique to construct accurate approximants to a given set of data or a given function by means of a local approach. A quasi-interpolant is usually obtained as a linear combination of a given system of blending functions that form a convex partition of unity and possess a small local support. These properties ensure both numerical stability and local control of the constructed approximant.

Hierarchical B-splines have been introduced in [1] as an accumulation of tensor-product B-splines with nested knot vectors. They are designed to do local refinement, which is not possible in the case of classical tensor-product splines. Hierarchical spline spaces are of interest in different areas ranging from approximation theory to geometric modeling and isogeometric analysis, a recent paradigm for the numerical treatment of partial differential equations.

In this talk we consider an alternative basis for the hierarchical spline space with an enhanced set of properties compared to the classical hierarchical B-spline basis. The so-called truncated hierarchical basis forms a convex partition of unity, its elements are more locally supported than those of the classical hierarchical basis, and it is a strongly stable basis with respect to the number of refinement levels [2,3].

We discuss a general approach to construct quasi-interpolants in hierarchical spaces expressed in terms of the truncated hierarchical basis [4]. The main ingredient is the property of preservation of coefficients of the truncated hierarchical basis representation. Thanks to this property, the construction of the hierarchical quasi-interpolant is basically effortless. It is sufficient to consider a quasi-interpolant in each space associated with a particular level in the hierarchy, which will be referred to as a one-level quasi-interpolant. Then, the coefficients of the proposed hierarchical quasi-interpolant are nothing else than a proper subset of the coefficients of the one-level quasi-interpolants. No additional manipulations are required. Important properties – like polynomial reproduction – of the one-level quasi-interpolants are preserved in the hierarchical construction. Moreover, it is easy to construct hierarchical local projectors, and so to provide dual bases. We also discuss the local approximation order of the hierarchical quasi-interpolants, and we illustrate the effectiveness of the approach with some numerical examples.
References


