Symbol Approach in IgA Matrix Analysis:
from the Spectral Analysis
to the Design of Fast Solvers

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Isogeometric Analysis (IgA) is a novel but well established paradigm for solving problems governed by PDEs [1]. The key ingredient in IgA is to use the same basis functions to describe the geometry of the physical domain and to approximate the solution of the differential problem. In its original formulation, IgA is a Galerkin approach based on NURBS, because they are the undisputed standard in CAD systems.

Any discretization of a linear PDE, for some sequence of stepsizes $h$ tending to zero, leads to a sequence of linear systems

$$A_n x_n = b_n,$$

where $\dim(A_n)$ increases when $h$ tends to 0. This is what happens also in the IgA case. To properly face the solution of such linear systems, it is important to understand the spectral properties of the matrices $A_n$.

The spectral distribution of a sequence of matrices is a fundamental concept. Roughly speaking, saying that the sequence of matrices $A_n$ has a spectral distribution described by a function $f$ means that the eigenvalues of $A_n$ behave as a sampling of $f$ over an equispaced grid of its domain. In this case, the function $f$ is called the symbol of the sequence of matrices $A_n$.

In this presentation we discuss the case of stiffness matrices arising from the IgA process [4]. We compute and analyze their symbol by exploiting the properties of cardinal B-splines. We also illustrate how the information contained in the symbol can be applied to design/analyze optimal two-grid and multigrid methods, as well as multi-iterative methods involving the PCG as a smoother in the multigrid procedure [2,3].

\textbf{References}

