Bound states computation for the nonlinear Schrödinger equation

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In this talk we present a numerical method to compute the bound states associated to the initial value problem for the nonlinear Schrödinger equation

\[
\begin{aligned}
& iu_t + u_{xx} \pm 2|u|^2u = 0, \quad x \in \mathbb{R}, \quad t > 0 \\
& u(x,0) = u_0(x), \quad x \in \mathbb{R}
\end{aligned}
\]  (1)

where \(i\) denotes the imaginary unit, \(u = u(x,t)\) is the unknown potential, the subscripts \(x\) and \(t\) designate partial derivatives with respect to position and time, \(u_0 \in L^1(\mathbb{R})\) is the initial potential and the \(\pm\) sign depends on symmetry properties of \(u\).

The numerical method is based on the following steps:

1. the computation of the so-called Marchenko kernels by solving Volterra integral equations on unbounded domains;
2. the computation of the Fourier transforms of the reflection coefficients by means of structured Fredholm integral equations of the second kind;
3. the identification of bound states which represent the parameters of a monomial-exponential sum [1].

In the talk we also show some numerical tests which confirm the effectiveness of the method.

References