Partition of unity interpolation using stable kernel-based techniques

R. Cavoretto$^a$, S. De Marchi$^b$, A. De Rossi$^a$, E. Perracchione$^a$, G. Santin$^b$

$^a$ Department of Mathematics “Giuseppe Peano”, University of Torino, Italy
$^b$ Department of Mathematics, University of Padova, Italy
roberto.cavoretto@unito.it, demarchi@math.unipd.it,
alessandra.derossi@unito.it, emma.perracchione@unito.it,
gsantin@math.unipd.it

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Considering the state of the art [1-4,7], we propose a new method for multivariate approximation which allows to interpolate large scattered data sets stably, accurately and with a relatively low computational cost.

The interpolant we consider is expressed as a linear combination of some basis or kernel functions. Focusing on Radial Basis Functions (RBFs), the partition of unity is performed by blending RBFs as local approximants and using locally supported weight functions. With this approach a large problem can be decomposed into many small problems, and therefore in the approximation process we could work with a big number of nodes (cf. [6]).

However, in some cases local approximants and consequently also the global one, may suffer from instability due to ill-conditioning of the interpolation matrices. This is directly connected to the order of smoothness of the basis function and to the node distribution. It is well-known that the stability depends on the flatness of the RBF. Specifically, for a flat basis function the condition number of the interpolation (or kernel) matrix might be quite severe, while a peaked one can be used to improve the conditioning but the accuracy of the fit gets worse. For this reason, the recent research is moved to the study of more stable bases. For particular RBFs, such as Gaussians, techniques allowing to stably and accurately compute the interpolant have already been designed, [5]. A different and more general approach, consisting in computing stable bases for a wider family of radial kernels via a truncated Singular Value Decomposition (SVD), was presented in [4].

Basically, the partition of unity method is usually applied by blending local RBF interpolants and using compactly supported weight functions. Here, instead, following [3], for each partition of unity subdomain a stable RBF basis is computed in order to solve the local interpolation problem. Consequently, since the local approximation order is preserved for the global fit, the interpolant will result more stable and accurate. Moreover, in terms of accuracy, the benefits coming from the use of such stable bases are more significant in a local approach than in a global one. In fact, generally, while in the global case a large number of truncated terms of the SVD must be dropped to preserve stability, a local par-
tition of unity technique requires only few terms are eliminated, thus enabling the method to be much more accurate.

Concerning the computational complexity of the algorithm, the use of the so-called kd-tree space partitioning data structure, which successfully works in any space dimension, enables us to efficiently organize points among the different subdomains, [1]. Then, for each subdomain a local RBF problem is solved with the use of a stable basis. The main and truly high cost, involved in this step, is the computation of the SVD. To avoid this drawback, techniques based on Krylov space methods are employed, since they turn out to be really effective (cf. [3]). A complexity analysis supports our findings.

Extensive numerical tests carried out with RBFs of different orders of smoothness, both globally and compactly supported, show the performance of the method on various data sets.

References