Geometrically continuous piecewise quasi
Chebyshevian splines

C. V. Beccari, G. Casciola
University of Bologna, Italy
carolina.beccari2@unibo.it, giulio.casciola@unibo.it

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Quasi extended Chebyshev spaces (QEC-spaces for short) represent the largest class of spaces of sufficient regularity which are suitable for design [1]. In particular, they contain the well-known extended Chebyshev spaces as a special instance (and therefore can represent transcendental functions) as well as the so called variable degree polynomial spaces [2], that are very effective in handling shape-preserving approximations.

Whereas it is natural to define quasi Chebyshevian splines as piecewise functions having sections in the same QEC-space, even more interesting is to consider piecewise quasi Chebyshevian splines, in which different sections may be in different QEC-spaces. In the most general setting, the continuity conditions between adjacent spline segments are of geometric type, namely they are expressed in terms of connection matrices, linking the appropriate number of either ordinary or generalized derivatives. Therefore, geometrically continuous piecewise quasi Chebyshevian splines combine the local nature of splines, the diversity of shape effects provided by the wide range of known QEC-spaces and the enhanced flexibility yield by the connection matrices.

Given an arbitrary partition of a closed real interval, a sequence of QEC-spaces and a sequence of connection matrices, we are interested in determining whether or not the resulting spline space has all the features that make it suitable for geometric design and approximation purposes.

To this aim, by extending to the geometric setting an approach which we recently proposed for parametrically continuous piecewise quasi Chebyshevian splines [3], we present a practical criterion and an effective numerical procedure to determine whether the space in question admits a totally positive B-spline type basis. Furthermore we illustrate a general methodology to explicitly construct and efficiently evaluate such basis.

References