Implementation and applications of Shanks transformation in a vector space

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When a sequence of numbers is slowly converging, it can be transformed into a new sequence which, under some assumptions, could converge faster to the same limit. One of the most well-known sequence transformation is Shanks’ transformation [3] which can be recursively implemented by the ε-algorithm of Wynn [4]. This transformation and this algorithm have been extended to sequence of vectors by Wynn [5] but they lack of an algebraic theory. Forty years ago, two different extensions to sequences of elements of a topological vector space $E$ were given Brezinski [1], together with their algebraic properties. However, the Brezinski’s algorithms require the use of linear functionals which are changing at each step.

In this talk, we present new algorithms for implementing these topological Shanks transformations [2]. They no longer require the manipulation of elements of the algebraic dual space of $E$, nor they use the duality product into the rules of the algorithms, they need the storage of less elements of $E$, and the numerical stability is improved. They also allow us to prove convergence and acceleration results for some types of sequences, results which were out of our reach with the previous algorithms.

These algorithms will be then applied to iterative methods for the solution of systems of nonlinear and matrix equations, and to the computation of matrix functions. Some examples on image reconstruction are also shown.

This is a joint work with C. Brezinski.

References